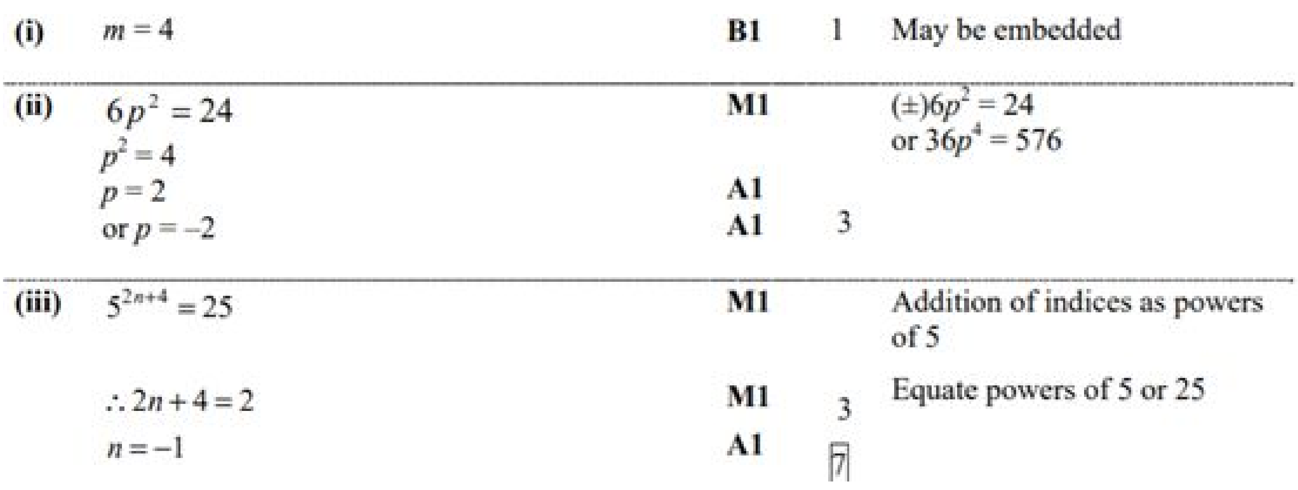
Inspired Learning

1.	$81^{\frac{3}{2}} = (81^{\frac{1}{2}})^3 = 9^3$ or $81^{\frac{3}{2}} = (81^3)^{\frac{1}{2}} = (531441)^{\frac{1}{2}}$	M1	
	$=729$	A1	
		(2)	
	$(4x^{-\frac{1}{2}})^2 = 16x^{-\frac{2}{2}}$ or $\frac{16}{x}$ or equivalent	M1	
	$x^2(4x^{-\frac{1}{2}})^2 \quad = 16x$	A1	
		(2) (4 marks)	
2.	**(i)** $n = -2$	**B1** $\boxed{1}$	
	(ii) $n = 3$	**B1** $\boxed{1}$	
	(iii)	**M1**	$\sqrt{4^3}$ or $64^{\frac{1}{2}}$ or $\left(4^{\frac{1}{2}}\right)^3$ or $(4^3)^{\frac{1}{2}}$ or $4\times\sqrt{4}$ with brackets correct if used
	$n = \frac{3}{2}$	**A1** $\boxed{2}$	

3.	**(a)**	$8^{\frac{1}{3}}=2$ or $8^{5}=32768$	A correct attempt to deal with the $\frac{1}{3}$ or the 5. $8^{\frac{1}{3}}=\sqrt[3]{8}$ or $8^{5}=8\times8\times8\times8\times8$	M1
		$\left(8^{\frac{5}{3}}=\right)$ 32	Cao	A1
		A correct answer with no working scores full marks		
		Alternative $8^{\frac{5}{3}}=8\times8^{\frac{2}{3}}=8\times2^{2}=$ M1 (Deals with the 1/3) $=32$ A1		
				(2)
	(b)	$\left(2x^{\frac{1}{2}}\right)^{3}=2^{3}x^{\frac{3}{2}}$	One correct power either 2^{3} or $x^{\frac{3}{2}}$. $\left(2x^{\frac{1}{2}}\right)\times\left(2x^{\frac{1}{2}}\right)\times\left(2x^{\frac{1}{2}}\right)$ on its own is not sufficient for this mark.	M1
		$\frac{8x^{\frac{3}{2}}}{4x^{2}}=2x^{-\frac{1}{2}}$ or $\frac{2}{\sqrt{x}}$	M1: Divides coefficients of x **and** subtracts their powers of x. **Dependent on the previous M1** A1: Correct answer	dM1A1
		Note that unless the power of x implies that they have subtracted their powers you would need to see evidence of subtraction. E.g. $\frac{8x^{\frac{3}{2}}}{4x^{2}}=2x^{\frac{1}{2}}$ would score dM0 unless you see some evidence that 3/2 – 2 was intended for the power of x.		
		Note that there is a misconception that $\frac{\left(2x^{\frac{1}{2}}\right)^{3}}{4x^{2}}=\left(\frac{2x^{\frac{1}{2}}}{4x^{2}}\right)^{3}$ - this scores 0/3		
				(3)
				[5]

4



(i)	$m = 4$	**B1**	1	May be embedded
(ii)	$6p^2 = 24$ $p^2 = 4$	**M1**		$(\pm)6p^2 = 24$ or $36p^4 = 576$
	$p = 2$	**A1**		
	or $p = -2$	**A1**	3	
(iii)	$5^{2n+4} = 25$	**M1**		Addition of indices as powers of 5
	$\therefore 2n + 4 = 2$	**M1**	3	Equate powers of 5 or 25
	$n = -1$	**A1**	$\boxed{7}$	

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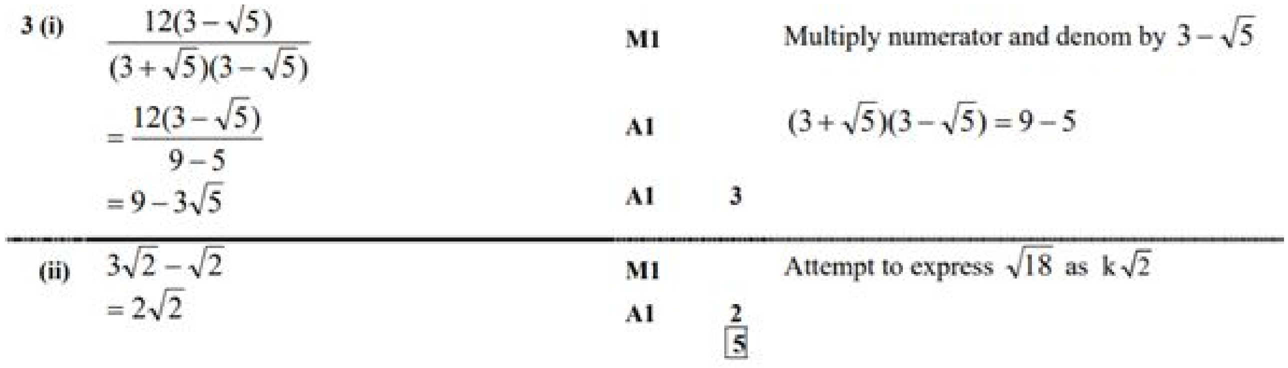

3 (i)	$\dfrac{12(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$	**M1**		Multiply numerator and denom by $3-\sqrt{5}$
	$= \dfrac{12(3-\sqrt{5})}{9-5}$	**A1**		$(3+\sqrt{5})(3-\sqrt{5}) = 9-5$
	$= 9 - 3\sqrt{5}$	**A1**	**3**	
(ii)	$3\sqrt{2} - \sqrt{2}$	**M1**		Attempt to express $\sqrt{18}$ as $k\sqrt{2}$
	$= 2\sqrt{2}$	**A1**	**2** $\boxed{5}$	

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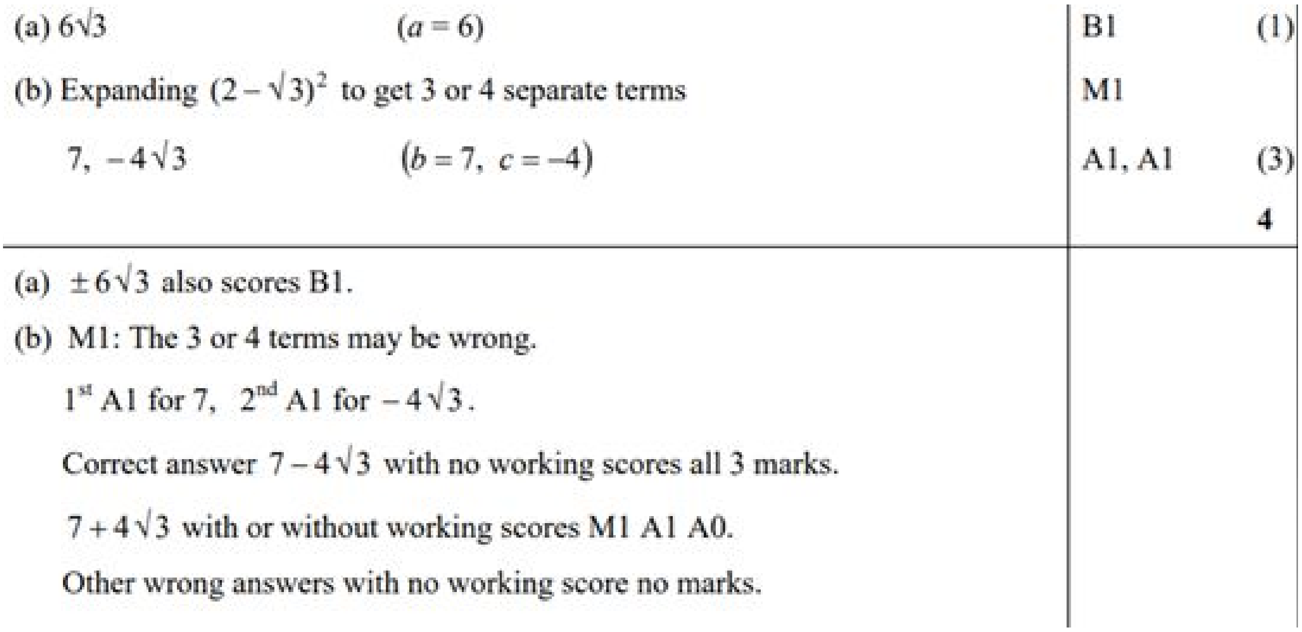


(a) $6\sqrt{3}$	$(a = 6)$	B1	(1)
(b) Expanding $(2-\sqrt{3})^2$ to get 3 or 4 separate terms		M1	
$7,\ -4\sqrt{3}$	$(b = 7,\ c = -4)$	A1, A1	(3)
			4

(a) $\pm 6\sqrt{3}$ also scores B1.

(b) M1: The 3 or 4 terms may be wrong.

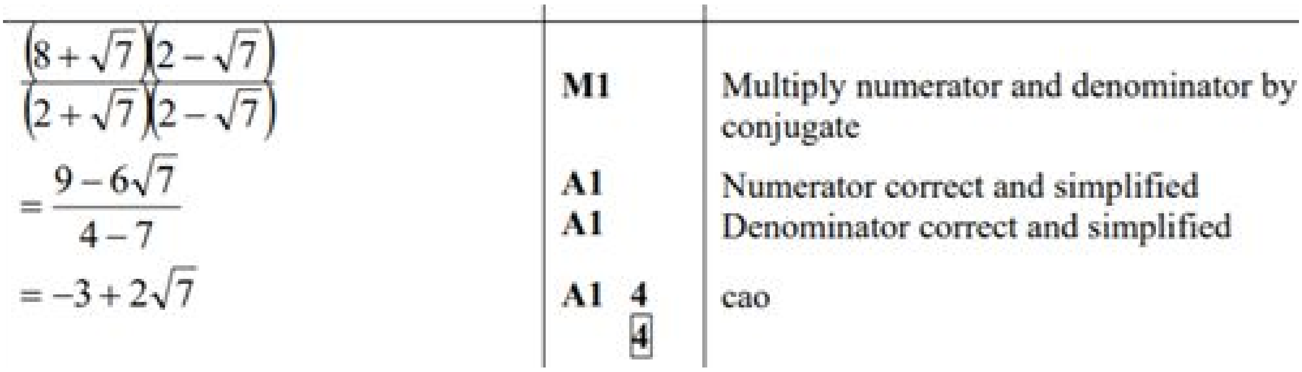
1st A1 for 7, 2nd A1 for $-4\sqrt{3}$.

Correct answer $7-4\sqrt{3}$ with no working scores all 3 marks.

$7+4\sqrt{3}$ with or without working scores M1 A1 A0.

Other wrong answers with no working score no marks.

7



$\frac{(8+\sqrt{7})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})}$	**M1**		Multiply numerator and denominator by conjugate
$=\frac{9-6\sqrt{7}}{4-7}$	**A1** **A1**		Numerator correct and simplified Denominator correct and simplified
$=-3+2\sqrt{7}$	**A1**	**4** $\boxed{4}$	cao

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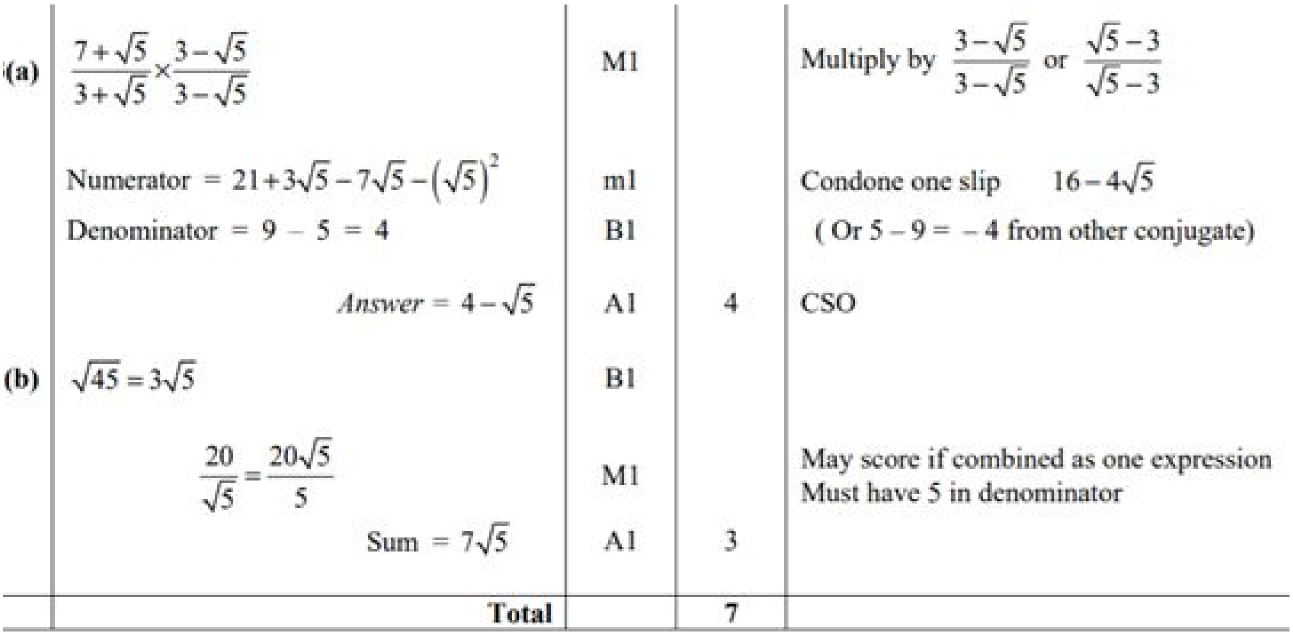

(a)	$\frac{7+\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}$	M1		Multiply by $\frac{3-\sqrt{5}}{3-\sqrt{5}}$ or $\frac{\sqrt{5}-3}{\sqrt{5}-3}$
	Numerator $=21+3\sqrt{5}-7\sqrt{5}-(\sqrt{5})^2$	m1		Condone one slip $16-4\sqrt{5}$
	Denominator = 9 − 5 = 4	B1		(Or 5 − 9 = − 4 from other conjugate)
	Answer = $4-\sqrt{5}$	A1	4	CSO
(b)	$\sqrt{45}=3\sqrt{5}$	B1		
	$\frac{20}{\sqrt{5}}=\frac{20\sqrt{5}}{5}$	M1		May score if combined as one expression Must have 5 in denominator
	Sum = $7\sqrt{5}$	A1	3	
	Total		**7**	

9	**5** **Method 1**	$x\sqrt{8}+10=\dfrac{6x}{\sqrt{2}}$	
		$\times\sqrt{2}\Rightarrow x\sqrt{16}+10\sqrt{2}=6x$	M1,A1
		$4x+10\sqrt{2}=6x\Rightarrow 2x=10\sqrt{2}$ $x=5\sqrt{2}$ or $a=5$ and $b=2$	M1A1
			(4)
	5 **Method 2**	$x\sqrt{8}+10=\dfrac{6x}{\sqrt{2}}$	
		$2\sqrt{2}x+10=3\sqrt{2}x$	M1A1
		$\sqrt{2}x=10\Rightarrow x=\dfrac{10}{\sqrt{2}}=\dfrac{10\sqrt{2}}{2},=5\sqrt{2}$ oe	M1,A1
			(4)

10	**(a)**	$\dfrac{5+\sqrt{7}}{3-\sqrt{7}}\times\dfrac{3+\sqrt{7}}{3+\sqrt{7}}$	M1		
		Numerator = $15+5\sqrt{7}+3\sqrt{7}+7$	m1		Condone one error or omission
		Denominator = $9-7$ $(=2)$	B1		Must be seen as the denominator
		(Answer =) $11+4\sqrt{7}$	A1	4	
	(b)	$\left(2\sqrt{5}\right)^2=20$ *or* $\left(3\sqrt{2}\right)^2=18$	B1		Either correct
		their $\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2$	M1		Condone missing brackets and x^2
		$(x^2=20-18)$			$x^2=2\Rightarrow$ Bl, Ml
		$(\Rightarrow x=)\sqrt{2}$	A1	3	$\pm\sqrt{2}$ scores A0 Answer only of 2 scores B0, M0 Answer only of $\sqrt{2}$ scores 3 marks
		Total		**7**	

11	**(a)**	$b^2-4ac<0 \Rightarrow$ e.g. $4^2-4(p-1)(p-5)<0$ or $0>4^2-4(p-1)(p-5)$ or $4^2<4(p-1)(p-5)$ or $4(p-1)(p-5)>4^2$	M1: Attempts to use b^2-4ac with at least two of a, b or c correct. May be in the quadratic formula. Could also be, for example, comparing or equating b^2 and $4ac$. Must be considering the given quadratic equation. Inequality sign not needed for this M1.There must be no x terms. A1: For a correct un-simplified **inequality** that is not the given answer	M1A1
		$4<p^2-6p+5$		
		$p^2-6p+1>0$	Correct solution with **no** errors that includes an expansion of $(p-1)(p-5)$	A1*
				(3)
	b)	$p^2-6p+1=0 \Rightarrow p=...$	For an attempt to solve $p^2-6p+1=0$ (**not <u>their</u> quadratic**) leading to 2 solutions for p (do not allow attempts to **factorise** – must be using the quadratic formula or completing the square)	M1
		$p=3\pm\sqrt{8}$	$p=3\pm2\sqrt{2}$ or any equivalent correct expressions e.g. $p=\dfrac{6\pm\sqrt{32}}{2}$ (May be implied by their inequalities) Discriminant must be a single number not e.g. 36 - 4	A1
		Allow the M1A1 to score anywhere for solving the given quadratic		
		$p<3-\sqrt{8}$ or $p>3+\sqrt{8}$	M1: Chooses outside region – **not dependent on the previous method mark** A1: $p<3-\sqrt{8}$, $p>3+\sqrt{8}$ or equivalent e.g. $p<\dfrac{6-\sqrt{32}}{2}$, $p>\dfrac{6+\sqrt{32}}{2}$ $(-\infty,\ 3-\sqrt{8})\cup(3+\sqrt{8},\ \infty)$ Allow “,”, “or” or a space between the answers but do **not** allow $p<3-\sqrt{8}$ **and** $p>3+\sqrt{8}$ (this scores M1A0) **Apply ISW if necessary.**	M1A1
		A correct solution to the quadratic followed by $p>3\pm\sqrt{8}$ **scores M1A1M0A0**		
		$3+\sqrt{8}<p<3-\sqrt{8}$ scores M1A0		
		Allow candidates to use x rather than p but must be in terms of p for the final A1		
				(4)
				(7 marks)

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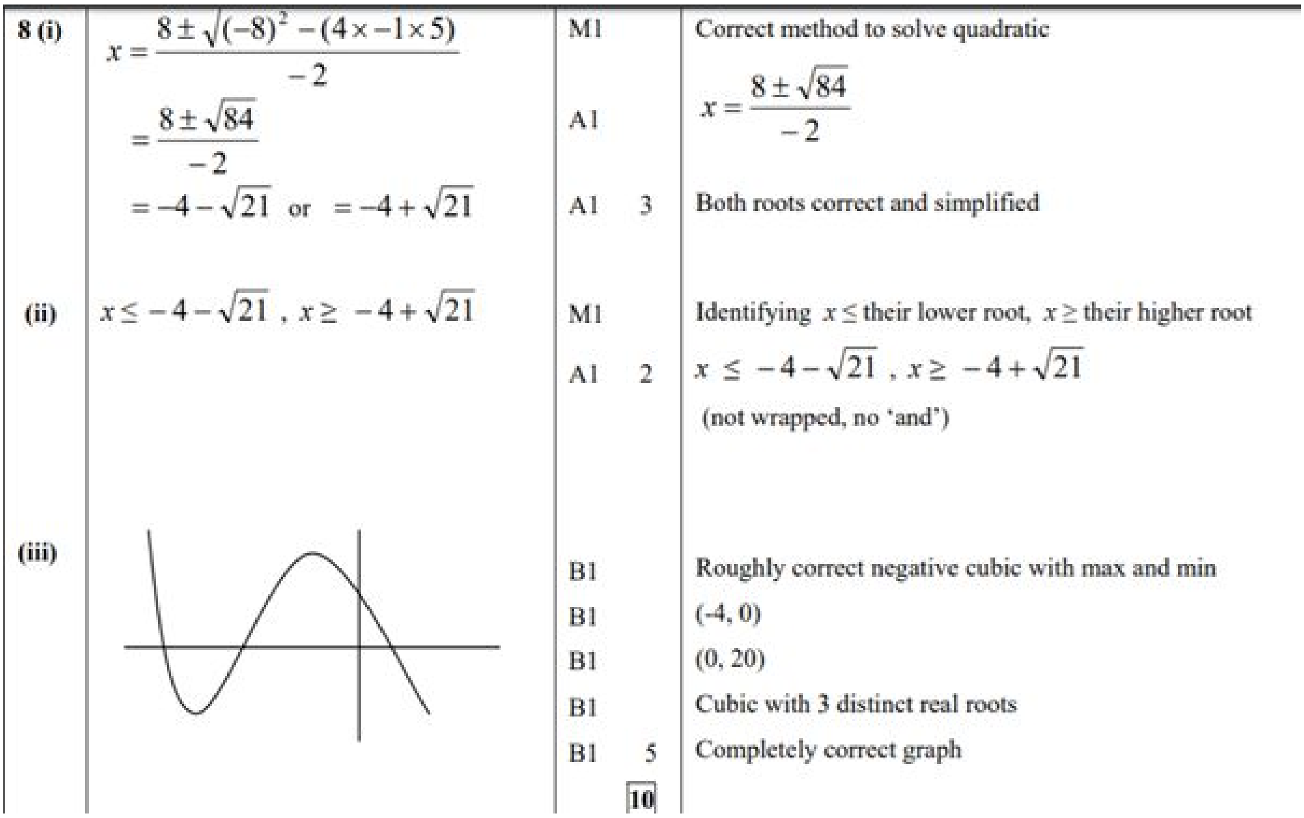
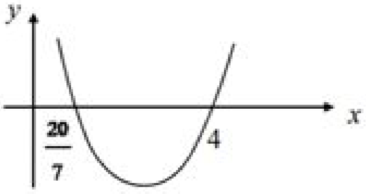


8 (i)	$x = \dfrac{8 \pm \sqrt{(-8)^2 - (4 \times -1 \times 5)}}{-2}$	M1		Correct method to solve quadratic
	$= \dfrac{8 \pm \sqrt{84}}{-2}$	A1		$x = \dfrac{8 \pm \sqrt{84}}{-2}$
	$= -4 - \sqrt{21}$ or $= -4 + \sqrt{21}$	A1	3	Both roots correct and simplified
(ii)	$x \leq -4 - \sqrt{21}$, $x \geq -4 + \sqrt{21}$	M1		Identifying $x \leq$ their lower root, $x \geq$ their higher root
		A1	2	$x \leq -4 - \sqrt{21}$, $x \geq -4 + \sqrt{21}$ (not wrapped, no 'and')
(iii)		B1		Roughly correct negative cubic with max and min
		B1		(-4, 0)
		B1		(0, 20)
		B1		Cubic with 3 distinct real roots
		B1	5	Completely correct graph
			$\boxed{10}$	

13	**(a)**	$(k-2)^2-4\times(2k-7)(k-3)$	M1		discriminant – condone one slip –condone omission of brackets
		$k^2-4k+4-4(2k^2-6k-7k+21)$	A1		
		"their" $-7k^2+48k-80\geqslant 0$	B1		real roots condition ; $f(k)\geqslant 0$ must appear before final line
		$7k^2-48k+80\leqslant 0$	A1cso	4	**AG** (all working correct with no missing brackets etc)
	(b)	$7k^2-48k+80=(7k-20)(k-4)$	M1		correct factors (or roots unsimplified) $\frac{48\pm\sqrt{64}}{14}$
		critical values are 4 and $\frac{20}{7}$	A1		accept $\frac{56}{14}, \frac{40}{14}$ etc here
			M1		sketch or sign diagram including values 
		$\frac{20}{7}\leqslant k\leqslant 4$ ***Take their final line as their answer***	A1cao	4	fractions must be simplified here
		Total		**8**	
		TOTAL		**75**	
14		$k=x^2$	**M1***		Use a substitution to obtain a quadratic or factorise into 2 brackets each containing x^2
		$4k^2+3k-1=0$			
		$(4k-1)(k+1)=0$	**M1 dep**		Correct method to solve a quadratic
		$k=\frac{1}{4}$ (or $k=-1$)	**A1**		
		$x=\pm\frac{1}{2}$	**M1** **A1**	**5** $\boxed{5}$	Attempt to square root to obtain x $\pm\frac{1}{2}$ and no other values

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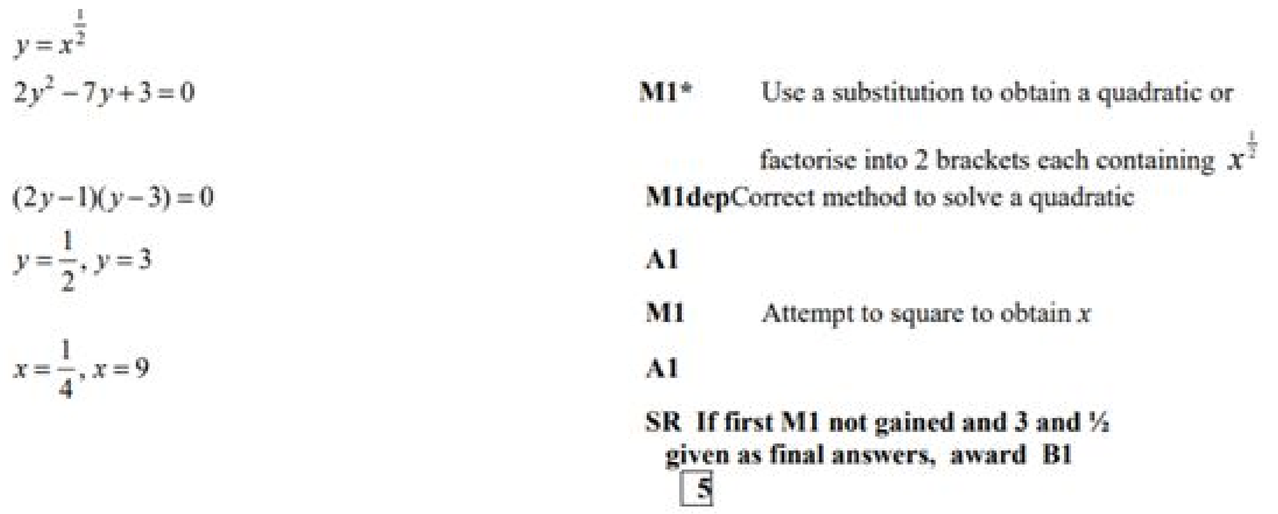

$y = x^{\frac{1}{2}}$		
$2y^2 - 7y + 3 = 0$	**M1***	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{2}}$
$(2y-1)(y-3) = 0$	**M1dep**	Correct method to solve a quadratic
$y = \frac{1}{2}, y = 3$	**A1**	
	M1	Attempt to square to obtain x
$x = \frac{1}{4}, x = 9$	**A1**	
	SR If first M1 not gained and 3 and ½ given as final answers, award B1 $\boxed{5}$	

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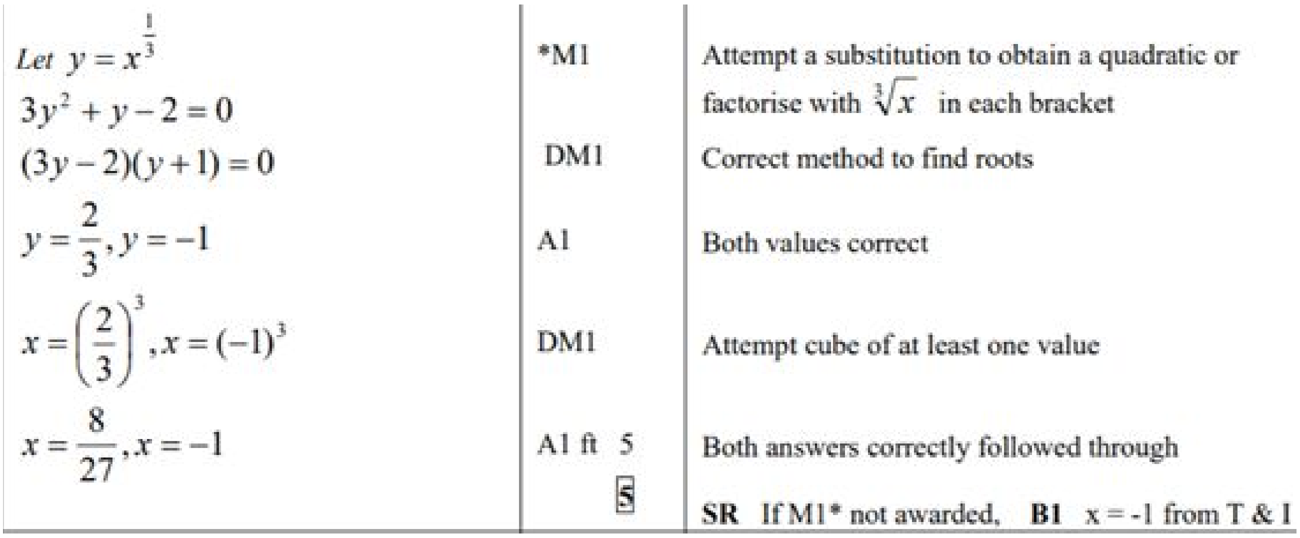

Let $y = x^{\frac{1}{3}}$ $3y^2 + y - 2 = 0$	*M1	Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket
$(3y-2)(y+1) = 0$	DM1	Correct method to find roots
$y = \frac{2}{3}, y = -1$	A1	Both values correct
$x = \left(\frac{2}{3}\right)^3, x = (-1)^3$	DM1	Attempt cube of at least one value
$x = \frac{8}{27}, x = -1$	A1 ft 5	Both answers correctly followed through
	$\boxed{5}$	**SR** If M1* not awarded, **B1** x = -1 from T & I

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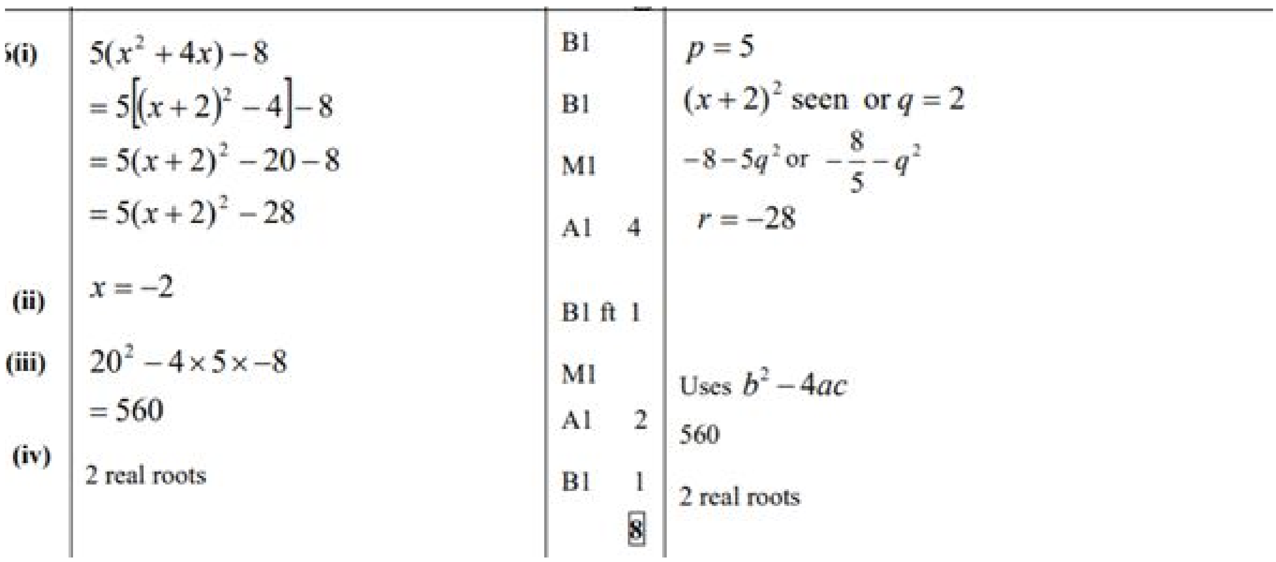

(i)	$5(x^2 + 4x) - 8$	B1	$p = 5$
	$= 5[(x+2)^2 - 4] - 8$	B1	$(x+2)^2$ seen or $q = 2$
	$= 5(x+2)^2 - 20 - 8$	M1	$-8 - 5q^2$ or $-\frac{8}{5} - q^2$
	$= 5(x+2)^2 - 28$	A1 4	$r = -28$
(ii)	$x = -2$	B1 ft 1	
(iii)	$20^2 - 4 \times 5 \times -8$	M1	Uses $b^2 - 4ac$
	$= 560$	A1 2	560
(iv)	2 real roots	B1 1	2 real roots
		$\boxed{8}$	

18		$5x^2+px-8=5(x-1)^2+r$	B1	$q=5$ (may be embedded on RHS)	
		$=5(x^2-2x+1)+r$ $=5x^2-10x+5+r$	B1	$p=-10$	
		$p=-10$ $r=-13$	M1	$-8=\pm q+r$ or $\frac{-p^2}{20}-8=r$	
			A1 **[4]**	$r=-13$	Allow from $p=10$
19		$2x^2-8x+8=26-3x$	**M1**	Attempt to eliminate x or y	Must be a clear attempt to reduce to one variable. Condone poor algebra for first mark.
		$2x^2-5x-18(=0)$	**A1**	Correct 3 term quadratic (not necessarily all in one side)	<u>If x eliminated:</u>
		$(2x-9)(x+2)(=0)$	**M1**	Correct method to solve quadratic	$y=2(\frac{26-y}{3}-2)^2$
		$x=\frac{9}{2},x=-2$	**A1**	x values correct	Leading to $2y^2-89y+800=0$
		$y=\frac{25}{2},y=32$	**A1** 5	y values correct	$(2y-25)(y-32)=0$ etc.
			5	**SR** If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**	
20	**(i)**	$2x^2-3x-5=\frac{-10x-11}{2}$	*M1	Substitute for x/y or attempt to get an equation in 1 variable only	or $10x+2(2x^2-3x-5)+11=0$
		$4x^2+4x+1=0$	A1	Obtain correct 3 term quadratic – could be a multiple e.g. $2x^2+2x+0.5=0$	If x is eliminated, expect $k(8y^2+48y+72)=0$
		$(2x+1)(2x+1)=0$	DM1	Correct method to solve resulting 3 term quadratic	
		$x=-\frac{1}{2}$	A1		**SC** If DM0 and $x=-\frac{1}{2}$ spotted
		$y=-3$	A1 **[5]**		**B1** for x value, **B1** for y value **B1** justifying only one root
	(ii)	Line is a tangent to the curve	B1√ **[1]**	**Must be consistent with their answers to their quadratic in (i).** **1 repeated root** – indicates one point. Accept tangent, meet at, intersect, touch etc. but do not accept cross **2 roots** – indicates meet at two points **0 roots** – indicates do not meet. Do not accept "do not cross"	**Follow through from their solution to (i)**

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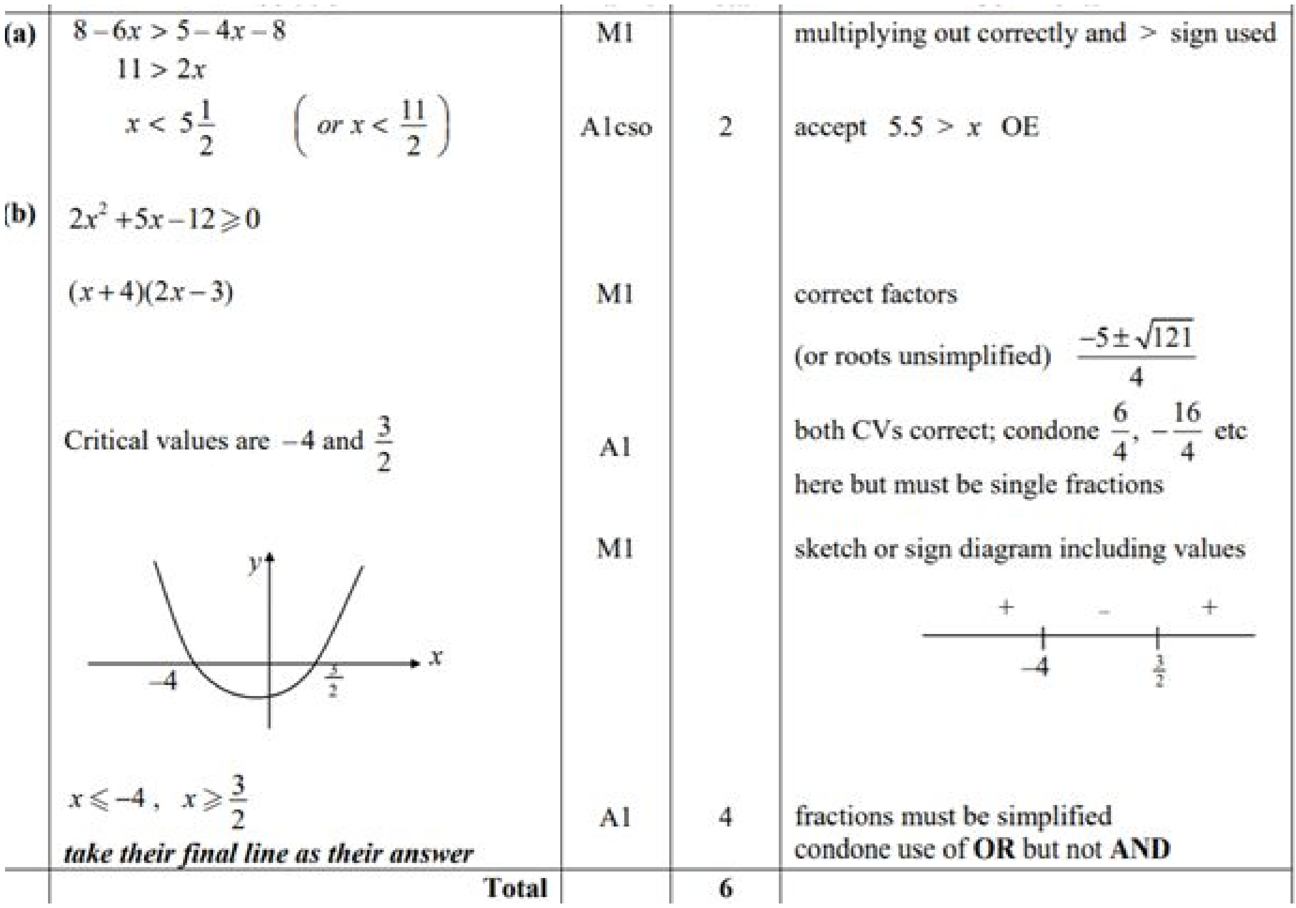

(a)	$8-6x>5-4x-8$ $11>2x$	M1		multiplying out correctly and $>$ sign used
	$x<5\frac{1}{2}$ $\left(or\ x<\frac{11}{2}\right)$	A1cso	2	accept $5.5>x$ OE
(b)	$2x^2+5x-12\geqslant 0$			
	$(x+4)(2x-3)$	M1		correct factors (or roots unsimplified) $\frac{-5\pm\sqrt{121}}{4}$
	Critical values are -4 and $\frac{3}{2}$	A1		both CVs correct; condone $\frac{6}{4}$, $-\frac{16}{4}$ etc here but must be single fractions
	y x –4 $\frac{3}{2}$	M1		sketch or sign diagram including values + - + –4 $\frac{3}{2}$
	$x\leqslant -4$, $x\geqslant\frac{3}{2}$ ***take their final line as their answer***	A1	4	fractions must be simplified condone use of **OR** but not **AND**
	Total		**6**	

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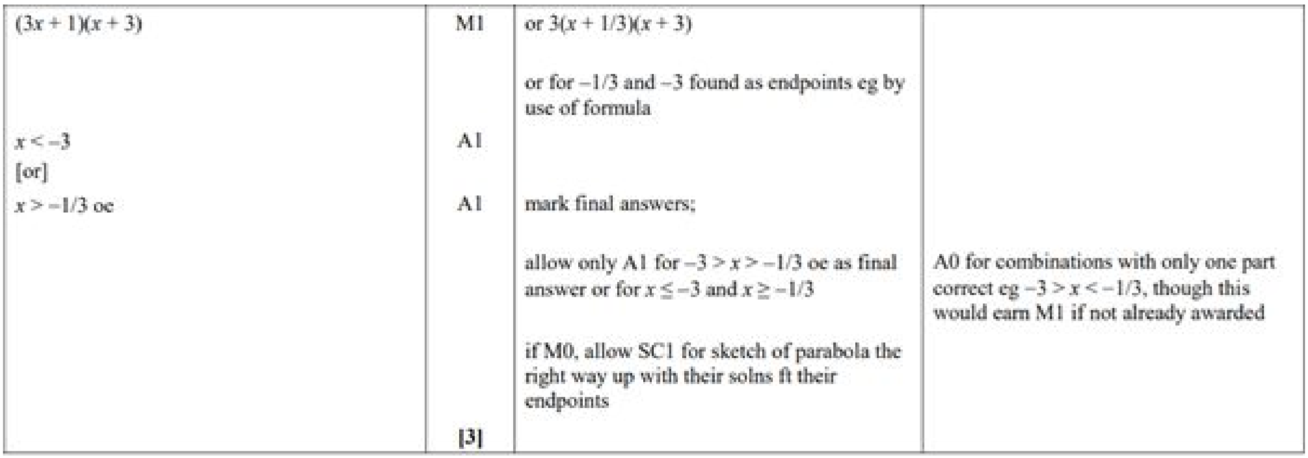

$(3x+1)(x+3)$	M1	or $3(x+1/3)(x+3)$ or for $-1/3$ and -3 found as endpoints eg by use of formula	
$x<-3$ [or]	A1		
$x>-1/3$ oe	A1	mark final answers; allow only A1 for $-3>x>-1/3$ oe as final answer or for $x\leq-3$ and $x\geq-1/3$ if M0, allow SC1 for sketch of parabola the right way up with their solns ft their endpoints	A0 for combinations with only one part correct eg $-3>x<-1/3$, though this would earn M1 if not already awarded
	[3]		

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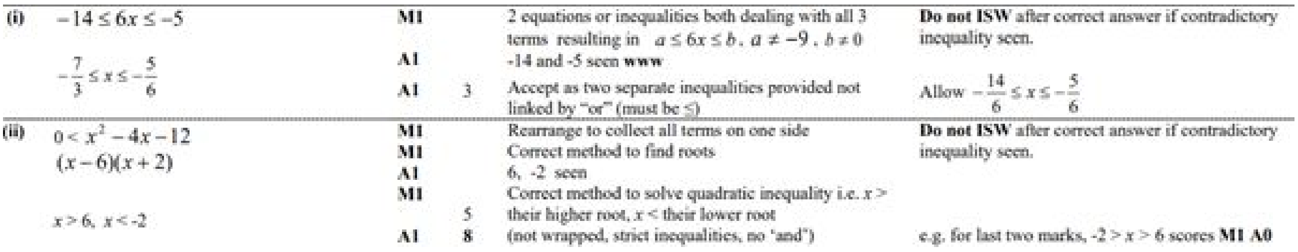
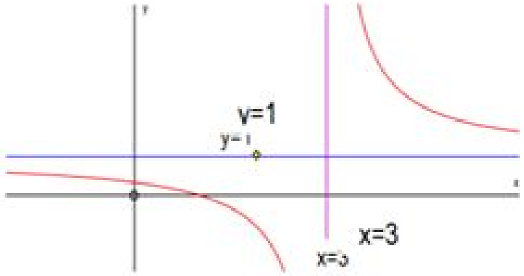
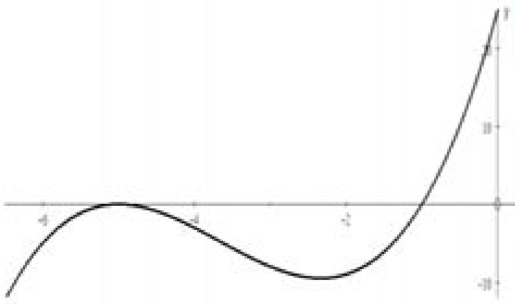
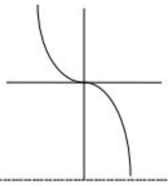


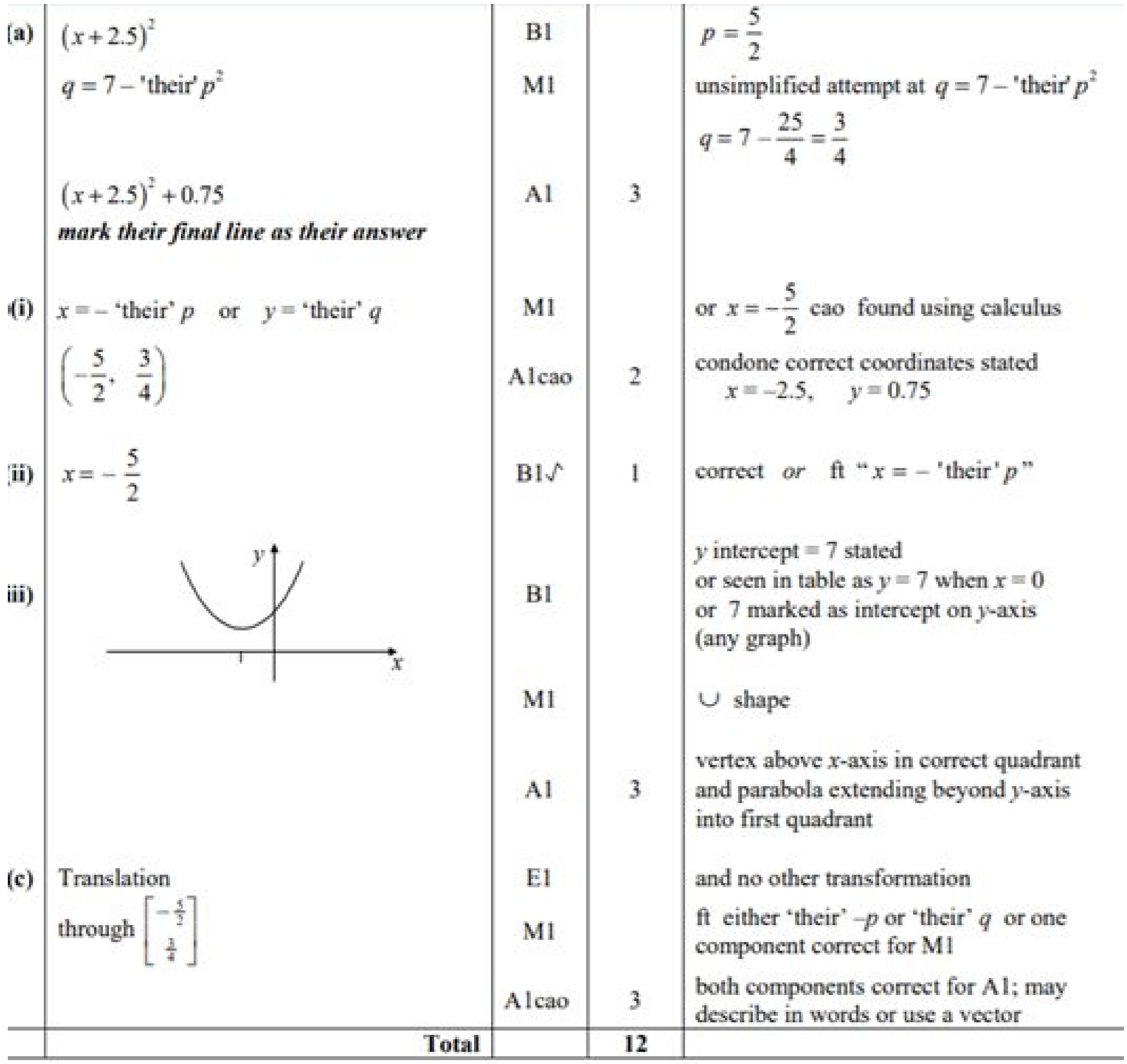
(i)	$-14\leq 6x\leq -5$	**M1**		2 equations or inequalities both dealing with all 3 terms resulting in $a\leq 6x\leq b$, $a\neq -9$, $b\neq 0$	**Do not ISW** after correct answer if contradictory inequality seen.
		A1		-14 and -5 seen **www**	
	$-\frac{7}{3}\leq x\leq -\frac{5}{6}$	**A1**	3	Accept as two separate inequalities provided not linked by "or" (must be $\leq$)	Allow $-\frac{14}{6}\leq x\leq -\frac{5}{6}$
(ii)	$0<x^2-4x-12$	**M1**		Rearrange to collect all terms on one side	**Do not ISW** after correct answer if contradictory inequality seen.
	$(x-6)(x+2)$	**M1**		Correct method to find roots	
		A1		6, -2 seen	
		M1	5	Correct method to solve quadratic inequality i.e. $x>$ their higher root, $x<$ their lower root	
	$x>6$, $x<-2$	**A1**	8	(not wrapped, strict inequalities, no 'and')	e.g. for last two marks, $-2>x>6$ scores **M1 A0**

24	6(a).	$P = 20x + 6$ o.e	B1	
		$20x + 6 > 40 \Rightarrow x >$	M1	
		$x > 1.7$	A1*	
				(3)
		Mark parts (b) and (c) together		
	(b)	$A = 2x(2x+1) + 2x(6x+3) = 16x^2 + 8x$	B1	
		$16x^2 + 8x - 120 < 0$	M1	
		Try to solve their $2x^2 + x - 15 = 0$ e.g. $(2x-5)(x+3) = 0$ so $x =$	M1	
		Choose inside region	M1	
		$-3 < x < \frac{5}{2}$ or $0 < x < \frac{5}{2}$ (as x is a length)	A1	
	(c)			(5)
		$1.7 < x < \frac{5}{2}$	B1cao	
				(1)
				(9 marks)

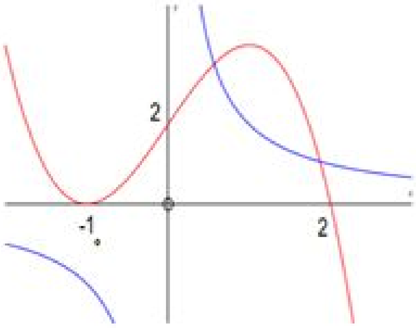
25	(a)			
		Correct shape with a single crossing of each axis	B1	
		$y = 1$ labelled or stated	B1	
		$x = 3$ labelled or stated	B1	
				(3)
	(b)	Horizontal translation so crosses the x-axis at $(1, 0)$	B1	
		New equation is $(y =)\dfrac{x \pm 1}{(x \pm 1) - 2}$	M1	
		When $x = 0$ $y =$	M1	
		$= \frac{1}{3}$	A1	
				(4) 7

26	**8** **a)**		Horizontal translation – does **not** have to cross the y-axis on the right but must at least reach the x-axis.	B1
			Touching at (-5, 0). This could be stated anywhere or -5 could be marked on the x-axis. Or (0, -5) **marked in the correct place.** Be fairly generous with 'touching' if the intention is clear.	B1
			The right hand tail of their cubic shape crossing at (-1, 0). This could be stated anywhere or -1 could be marked on the x-axis. Or (0, -1) **marked in the correct place.** The curve must **cross** the x-axis and not stop at -1.	B1
				(3)
	b)	$(x+5)^2(x+1)$	Allow $(x+3+2)^2(x-1+2)$	B1
				(1)
	c)	When $x = 0, y = 25$	M1: Substitutes $x = 0$ into their expression in **part (b)** which is not f(x). This may be implied by their answer. Note that the question asks them to use part (b) but allow independent methods.	M1 A1
			A1: $y = 25$ (Coordinates not needed)	
		If they expand incorrectly prior to substituting $x = 0$, score M1 A0 **NB f(x + 2) = $x^3 + 11x^2 + 35x + 25$**		

27	**5 (i)**		**M1**		Negative cubic through (0, 0) (may have max and min)	Must be continuous. Allow slight curve towards or away from y-axis at one end, but not both.
			A1	**2**	Must have reasonable rotational symmetry. Cannot be a finite "plot". Allow negative gradient at origin. Correct curvature at both ends.	
	(ii)	$y = -(x-3)^3$	**M1**		$\pm(x-3)^3$ seen	
			A1	**2**	or $y = (3-x)^3$	Must have "$y =$ " for A mark **SR** $y = -(x-3)^2$ **B1**
	(iii)	Stretch scale factor 5 parallel to y-axis	**B1** **B1**	**2** $\boxed{6}$	o.e. e.g. scale factor $\frac{1}{\sqrt[3]{5}}$ parallel to the x axis.	Allow "factor" for "scale factor" For "parallel to the y axis" allow "vertically", "in the y direction". **Do not accept** "in/on/across/up/along the y axis"



28	(a)	$(x+2.5)^2$	B1		$p=\frac{5}{2}$
		$q=7-$'their' p^2	M1		unsimplified attempt at $q=7-$'their' p^2 $q=7-\frac{25}{4}=\frac{3}{4}$
		$(x+2.5)^2+0.75$ ***mark their final line as their answer***	A1	3	
	(i)	$x=-$'their' p or $y=$'their' q	M1		or $x=-\frac{5}{2}$ cao found using calculus
		$\left(-\frac{5}{2},\ \frac{3}{4}\right)$	A1cao	2	condone correct coordinates stated $x=-2.5$, $y=0.75$
	(ii)	$x=-\frac{5}{2}$	B1√	1	correct *or* ft "$x=-$'their' p"
	iii)		B1		y intercept = 7 stated or seen in table as $y=7$ when $x=0$ or 7 marked as intercept on y-axis (any graph)
			M1		∪ shape
			A1	3	vertex above x-axis in correct quadrant and parabola extending beyond y-axis into first quadrant
	(c)	Translation	E1		and no other transformation
		through $\begin{bmatrix}-\frac{5}{2}\\ \frac{3}{4}\end{bmatrix}$	M1		ft either 'their' $-p$ or 'their' q or one component correct for M1
			A1cao	3	both components correct for A1; may describe in words or use a vector
		Total		**12**	

29	(a)	$(a=)\ (1+1)^2(2-1)=\underline{4}$ (1, 4) or $y=4$ is also acceptable	B1	(1)
	(b)			
		(i) Shape  or  anywhere	B1	
		Min at $(-1,0)$ … can be -1 on x-axis. Allow $(0,-1)$ if marked on the x-axis. Marked in the correct place, but 1, is B0.	B1	
		(2, 0) and (0, 2) can be 2 on axes	B1	
		(ii) Top branch in 1st quadrant with 2 intersections	B1	
		Bottom branch in 3rd quadrant (ignore any intersections)	B1	(5)
	(c)	(2 intersections therefore) $\underline{2}$ (roots)	B1ft	(1) **[7]**